INDIRECTLY SUPPORTED BRIDGES – RISK OF BRITTLE FAILURE?

Karel Thoma, Prof. Dr., Dept. of Civil Engineering, Lucerne University, Switzerland
Christoph Bueeler, dipl. Bauing. FH, Dept. of Civil Engineering, Lucerne University, Switzerland

ABSTRACT

For concrete bridge structures, current structural codes, like the Swiss Code SIA 262, require that in cases of indirect support conditions, force transfers need to be checked utilizing stress fields. This is necessary to guarantee that sufficient hanger reinforcement is provided at cross-section discontinuities. If the entire shear force is supported by hanger reinforcement, it is possible to investigate load path in the longitudinal and transverse girders independently of each other.

Today, although this code is in place, many older existing indirectly supported reinforced concrete bridge structures were designed with insufficient hanger reinforcement. Thus, for these older bridges it is necessary that interactions between longitudinal and transverse force flow be considered. The strength of the entire system is largely dependent on the arrangement of the reinforcement in the transverse beam and the resulting stress fields in the longitudinal and transverse beams.

Two topics, in regards to indirect support conditions, are investigated in this paper. First, the influence of insufficient hanger reinforcement on the strength of reinforced concrete bridges that are prestressed in longitudinal and/or transverse direction is demonstrated. Secondly, the risk of brittle failure is discussed. To conduct these investigations, stress field models (including the required examination of deformation capacities) are utilized. The theoretical findings are verified through a preliminary experimental test on an indirectly supported, longitudinally and transversely prestressed two-span beam.

Keywords: Bridge, Indirect support, Brittle failure, Strength reserve, Plasticity theory, Stress field models.
LITERATURE REVIEW

STRESS FIELD MODELS

Stress field models are based on the theory of plasticity. The idea of using truss models to calculate the internal forces of structural concrete beams was initially introduced by Ritter \(^1\) and Moersch \(^2\). Later, Drucker \(^3\) developed the first stress field models based on the theory of plasticity. Summaries on this method are presented in Thuerlimann et al. \(^4,5\), Nielsen \(^6\), Chen \(^7\), Marti \(^8\), Collins and Mitchell \(^9\) and Muttoni et al. \(^10\).

SHEAR TESTS ON INDIRECTLY SUPPORTED PRESTRESSED CONCRETE BEAMS

The first large scale experiments of indirectly supported prestressed concrete beams were conducted at the University of Stuttgart in the 1970s \(^11,12\). The force flow and the influence of the shear force at cross-section discontinuities were investigated for different reinforcement layouts. The researchers discovered that the entire shear force must be supported by hanger reinforcement in order to investigate the load path in the longitudinal and transverse beams independently of each other. Despite the experimental result, Leonhardt \(^13\) introduced a model to calculate the reinforcement requirements for indirect support conditions, which allows distributing the required hanger reinforcement around the cross section discontinuity as exemplified in Fig. 1 (a).

In the late 1990s, one of the largest bridges in Zurich, known as the “Europabruecke”, was evaluated in terms of strength. At this time, the indirect support of the bridge was found to be inadequate. As a result of this finding, Marti and Stoffel \(^14\) tested four large scale beams to investigate the force flow and the influence of the hanger reinforcement on the ultimate strength of indirectly supported bridges. Based on these experiments, Stoffel \(^15\) later introduced discontinuous stress field models describing the force flow for indirect support conditions, as shown in Fig 1 (b).

\[\text{Fig. 1 Indirect support; (a) model introduced by Leonhardt}^{13}\]
\[\text{(b) stress field models introduced by Stoffel}^{15}\]
DISCONTINUOUS STRESS FIELD MODELS

The discontinuous stress field models presented in this paper are based on the lower-bound theorem of the theory of plasticity. This theorem states that any load corresponding to a statically admissible stress field, at yield or below, is no higher than the ultimate load. Whereas it is possible to determine internal forces with a truss model, the calculation of stresses and dimensions as well as the proper detailing of the reinforcement is only possible with stress field models. Solving the conditions of equilibrium for the free-body diagram of Fig. 2 (a), the concrete compressive stresses, $\sigma_{c3}$, and the differential top chord forces, $dF_{sup}(x)/dx$, can be derived according to equation (1). In this equation, $\Theta$ denotes the angle of the compressive stress, $\sigma_{c3}$, $b_w$ denotes the web thickness, and $f_w$ represent the transverse stirrup forces.

$$\sigma_{c3} = \frac{q + f_w}{b_w \cdot \sin^2 \Theta} \cdot \frac{dF_{sup}}{dx} = -(q + f_w) \cdot \cot \Theta$$ (1)

PARALLEL STRESS FIELD

Under the assumption of a constant effective shear depth, $z$, and concrete compressive stress angle, $\theta_{const}$, solving equation (1) results in the parallel stress field illustrated in Fig. 2 (b). The compressive stress, $\sigma_{c3}$, is constant within the parallel stress field and the top chord forces, $F_{sup}(x)$, vary linearly according to equation (2).

$$\sigma_{c3} = \frac{q + f_{w,r}}{b_w} \cdot (1 + \cot^2 \Theta), \quad F_{sup}(x) = F_{sup}(x = 0) - (q + f_{w,r}) \cdot x \cdot \cot \Theta$$ (2)

CENTERED FAN

By applying equation (1) to the top stringer of a centered fan, equation (3) can be derived (see Fig. 2 (c)). While the angle of the compressive stress, $\Theta$, is constant within a parallel stress band, the angle $\Theta$ is not constant within a centered fan. Therefore, the top chord forces, $F_{sup}(x)$, and the concrete compressive stresses, $\sigma_{c3}(x, y)$, vary non-linearly. Given these findings, centered fans can be considered in practical design problems. It must be noted that the calculation of the nodal zone or the exact variation of the top chord forces is only necessary in special cases.

$$\sigma_{c3}(y, x) = \frac{q + f_{w,x}}{b_w} \cdot \left(1 + \left(\frac{x}{z}\right)^2\right) \cdot \frac{z}{y}, \quad F_{sup}(x) = F_{sup}(x = 0) - (q + f_w) \cdot \frac{x^2}{2} \cdot \frac{1}{z}$$ (3)

In equation (3), $z$ denotes the effective shear depth. Based on equation (1), it is possible to develop more complex stress fields (e.g., for prestressed concrete structures).
INDIRECT SUPPORT

BASIC CONCEPT OF INDIRECT SUPPORT

In the following section, the basic concept of indirect support is explained in terms of the numerical example illustrated in Fig. 3 (a). A single-span girder is subjected to a concentrated load $Q$. This longitudinal girder is indirectly supported by a transverse girder, which is also a single-span beam. The distribution of forces requires a load transfer from the longitudinal girder to the transverse girder at the cross section discontinuity $A$. The calculation of the stress field solution and the truss model is based on the “stringer cross-section model” illustrated in Fig. 3 (b). Given this modeling assumption, the effective shear depth, $z$, is constant over the longitudinal and the transverse girders.

Fig. 3 (c) and Fig. 3 (d) show two possible stress field solutions and the corresponding truss models. The stress field model according to Fig. 3 (c) requires concentrated hanger reinforcement at the cross section discontinuity $A$, as shown in Fig. 3 (e). If the entire shear force in $A$ is supported by hanger reinforcement, it is possible to investigate the force flow in the longitudinal and transverse girders independently of each other. If no additional hanger reinforcement is provided at the cross section discontinuity $A$, the stress field model and the truss model, as illustrated in Fig. 3 (d), are suitable solutions. To facilitate load transfer in $A$, parallel stress bands in the longitudinal and transverse girders are necessary. As a result of this discontinuous stress field model, no anchorage of the bottom chord force, $F_{inf}$, needs to be provided in $A$. However, longitudinal reinforcement distributed over the effective shear depth is required in the longitudinal girder (refer to Fig. 3 (f)). Combinations of these two discontinuous stress field models are possible, if hanger reinforcement is provided at the cross section discontinuity. The internal forces depend on the amount of hanger reinforcement provided at such discontinuity.

This example illustrates, that the ultimate strength of the entire system is largely dependent on the arrangement of the reinforcement (especially the hanger reinforcement) as well as the resulting stress field models in the transverse and longitudinal girders.
Fig. 3 Indirect support - numerical example: (a) geometry and loading; (b) stringer model; discontinuous stress field, truss model and resulting internal forces, (c) sufficient hanger reinforcement at cross-section discontinuity $A$, (d) no hanger reinforcement at cross-section discontinuity $A$. 
LARGE-SCALE EXPERIMENT

A large-scale experiment was carried out at the University of Lucerne in July of 2009. For a detailed discussion refer to Bueeler and Thoma. The test specimen reflected a continuous, single celled box girder in the region of a continuous support.

Fig. 4 Large-scale experiment; (a) experimental setup; (b) geometry of prestressed transverse girder; (c) geometry of prestressed longitudinal girder; (d) reinforcement layout.

The reinforcement layout of the two longitudinal girders was identical with the exception of the flexural tension reinforcement in the bottom chord (LG 1: 8 ø 8 mm; LG 2: 10 ø 14 mm). The shear reinforcement of the longitudinal and transverse girders was constant over the entire length, as depicted in Fig. 4. No additional hanger reinforcement was provided at the cross-section discontinuities. Furthermore, the internal prestress was provided by type 810 tendons (18 wires ø 7 mm) with a flat cladding tube.
During the first experimental phase, the test specimen was subjected to cyclic loading between 500 kN and 1000 kN which resulted in the decompression of the cross-section in the vicinity of the continuous support. A total of 60 load cycles were applied (see Fig. 5). During the second experimental phase, the applied load was increased to 635 kN enforcing a ductile bending failure of LG 1. Fig. 5 (a) shows the shear force versus deformation diagram of LG 1 at the cross section discontinuity, $A_1$. Fig. 6 (a) and Fig. 7(a) illustrate the plastic hinge and the crack-pattern at failure, respectively. Afterwards, the experimental setup was changed and the applied load was increased until the entire shear force at the cross section discontinuity, $A_2$, reached 863 kN resulting in a shear failure in the transverse girder. Fig. 5 (b) shows the shear force versus mid-span deflection diagram at discontinuity, $A_2$, of LG 2. Fig. 6 (b) and Fig. 7 (b) display the failure mechanism and crack-pattern of LG 2, respectively.

The experiment clearly illustrates that the ultimate load and the failure mechanism of an indirectly supported concrete structure strongly depend on the reinforcement arrangement and the detailing. Furthermore, the deformation capacity of the plastic hinge also largely depends on the reinforcement arrangement.

![Fig. 5 Shear force versus mid-span deformation diagram; (a) LG 1; (b) LG 2.](image1)

![Fig. 6 Failure mechanism; (a) ductile bending failure LG 1; (b) shear failure LG 2.](image2)
Fig. 7 Failure mechanism and crack pattern of transverse girder; (a) ductile bending failure LG 1; (b) shear failure LG 2.

STRESS FIELD SOLUTION FOR LONGITUDINAL GIRDER 1

Fig. 8 and Fig. 9 show a discontinuous stress field model for the experimental phase 2 and the failure mechanism of LG 1, respectively. For the plastic limit state analysis, the prestress was modeled as an applied load and it was based upon the effective crack pattern at failure. The assumed discontinuous stress field model for the transverse girder utilizes a combination of direct support transfer, centered fan and parallel stress-band actions. All shear reinforcement was taken into account, and the stirrups in the cross-section discontinuity were effective as hanger reinforcement with a total tension force of 187 kN. (3 stirrups ø 8 mm – 187 kN). Enforcing equilibrium at the cross-section discontinuity \( A \), results in the stress field model for the longitudinal girder. This is illustrated in Fig. 8. Considering the dead load, a bearing reaction of 309 kN results, which agrees well with the effective bearing reaction of 352 kN (including dead load). Therefore, the limit state analysis is in good agreement with the experimental results and the absolute error amounts to less then 15%.

Fig 8 Free-body diagrams of transverse girder and the corresponding discontinuous stress field model (stress field separated for prestress and mild reinforcement) [kN,m].
CONCLUSIONS

- The ultimate load of indirectly supported reinforced concrete structures strongly depends on the reinforcement arrangement (especially the hanger reinforcement at cross-section discontinuities).
- If the entire shear force in the cross-section discontinuities is supported by hanger reinforcement, it is possible to investigate the force flow in the longitudinal and transverse girders independently of each other.
- Limit state analysis, based on discontinuous stress fields, provides a good calculation of the ultimate load of indirectly supported reinforced concrete structures.
- The deformation capacity of plastic hinges at continuous supports (e.g. indirect supports) needs to be checked carefully to prevent brittle failure mechanisms.

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REFERENCES